V. Conclusions

We propose a new evolutionary direction operator for genetic algorithms. We solved a maximum value search problem of a two-dimensional multiple-peak function. The results show that the evolutionary direction operator is very effective. According to this operator, the evolutionary direction is determined simply, and thus computation time is reduced if compared with the gradient-based optimizers that require evaluation of gradients. This operator is expected to be applicable to almost every genetic algorithm easily because details of the target problem are not required.

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Quick Optimum Buckling Design of Axially Compressed, Fiber Composite Cylindrical Shells

Rolf Zimmermann*

DLR, German Aerospace Research Establishment, D-38022 Braunschweig, Germany

Nomenclature

A = membrane stiffness matrix
B = coupling stiffness matrix
D = bending stiffness matrix
M = moment vector per unit length
m = axial half-wave number of the deformation mode
N = force vector per unit length
n = circumferential wave number of the deformation mode
ε = reference surface strain vector
κ = curvature change vector

Introduction

LIGHTWEIGHT, thin-walled structures loaded by compression and/or shear may fail due to buckling. Laminates made from fiber-reinforced plastics (FRP) are promising candidates for use in such applications. Among them, axially compressed FRP cylindrical shells play an important role, both because of their application as real shells and because of their suitability to serve as specimens for basic buckling research. Such shells could be accomplished as stiffened, unstiffened, or sandwich shells. Which of these types will be chosen depends on real application. The decision to be made requires a comparison on the basis of optimum respective configurations. Thus, optimization of the different types of construction is a precondition for the choice of the best type, and quick optimum design is desirable. The present work considers unstiffened shells. As the failure is dominated by buckling, which on the other hand is strongly affected by laminate stiffnesses, laminate stacking is of main influence; thus, this should be considered in optimum design. But, in particular, fiber orientations as design variables require substantial expenditure of computer time, which impedes quick design. The application of simple rules might be the remedy to overcome this conflict. It is the main objective of this work to look for such simple rules for quick design. They will be derived from a database, which is generated by a great many of numerical optimization runs. The following sections on the structural model and optimization procedure describe how this database has been generated, whereas the results section reports on its evaluation leading to the simple rules for quick optimum design. The literature available, e.g., Refs. 1–3, merely considers subaspects of the problem. The work presented here is part of a much more extended study.[4,5]

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*Scientist, Dr.-Ing., Thermo-Resistant Structures, Institute of Structural Mechanics.

Structural Model

Figure 1 shows the cylinder considered, with radius R, length L, and compressive force F. The wall of the shell consists of layers with thicknesses $t_1, t_2, \ldots, t_p$, where the index 1 denotes the inner layer and the index p the outer one. The thicknesses of the layers are constant over the length. The thinnest unit is a tape, which consists of unidirectionally oriented fibers embedded in a matrix. The next thickest unit is an angle ply, formed from two tapes: one is arranged at $+\alpha_i$ to the axis of the shell and the other one at $-\alpha_i$. The index i denotes the layer to which the tapes belong. The thickness of this unit is t_0; the unit is called here basic unit. From now on, $+\alpha_i$ and $-\alpha_i$ will be considered as a single orientation α_i. Several of the basic units are coupled together resulting in the whole laminate, the thickness of which is $t_l = p \cdot t_0$. The parameter p is the number of basic units in the laminate and, as such, the maximum possible number of layers with different orientations α_i. Apart from considering an angle ply to be homogeneous over its thickness, no simplification is made in regard to stacking. In particular, symmetry of the entire laminate is not postulated. The mechanical characteristics of this setup are described by means of the stiffness matrix of the laminate, which is formulated assuming classical lamination theory,[6]

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon \\ \kappa \end{bmatrix} \tag{1}$$

The materials considered here are carbon fiber-reinforced plastics (CFRP) with $Q_{11} = 124446$ N/mm^2, $Q_{12} = 2802$ N/mm^2, $Q_{22} = 8771$ N/mm^2, and $Q_{66} = 5695$ N/mm^2 as stiffness properties of a unidirectional layer, and glass fiber-reinforced plastics (GFRP) with $Q_{11} = 47120$ N/mm^2, $Q_{12} = 3862$ N/mm^2, $Q_{22} = 13316$ N/mm^2, and $Q_{66} = 4300$ N/mm^2.

The model behind the buckling load analysis must allow fast computations, but it also should keep the main attributes affecting buckling. Thus, the classical bifurcation buckling formula for axially compressed, eccentrically orthotropic, shallow cylindrical shells will be used,[7] which can be written as follows:

$$\tilde{F} = 2\pi R \left\{ \min_{m,n} \left[\frac{1}{\beta^2} \left(S_1 + \frac{S_2^2}{S_3} \right) \right] \right\} \tag{2}$$

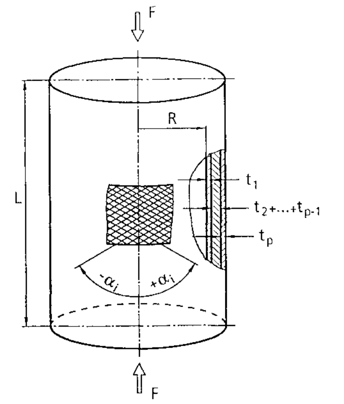

Fig. 1 Axially compressed FRP cylindrical shell.

with

$$S_1 = \tilde{D}_{11}\beta^4 + 2(\tilde{D}_{12} + 2\tilde{D}_{66})\beta^2\eta^2 + \tilde{D}_{22}\eta^4$$

$$S_2 = e_{21}\beta^4 + (e_{11} + e_{22} - 2e_{66})\beta^2\eta^2 + e_{12}\eta^4 + (1/R)\beta^2$$

$$S_3 = a_{22}\beta^4 + 2(a_{12} + 0.5a_{66})\beta^2\eta^2 + a_{11}\eta^4$$

$$\beta = m\pi/L, \qquad \eta = n/R$$

The terms $\tilde{D}_{jk}$, e_{jk}, and a_{jk} as elements of matrices $\tilde{\boldsymbol{D}}$, $\boldsymbol{e}$, and $\boldsymbol{a}$, respectively, are defined by the operations

$$\boldsymbol{a} = \boldsymbol{A}^{-1}, \qquad \boldsymbol{e} = \boldsymbol{A}^{-1}\boldsymbol{B}, \qquad \tilde{\boldsymbol{D}} = \boldsymbol{D} - \boldsymbol{B}\boldsymbol{A}^{-1}\boldsymbol{B}$$

m and n mark the axial half wave number and the circumferential wave number of the deformation mode, respectively. The buckling load $\tilde{F}$ results from minimization in regard to m and n. The minimum characterizes the buckling load, whereas the respective values of m and n describe the buckling mode.

Optimization Procedure

The optimization problem is formulated as buckling load maximization,

$$\max_{x}\{f(\boldsymbol{x}) \mid \boldsymbol{g}(\boldsymbol{x}) \le 0\} \tag{3}$$

with

$$f(\boldsymbol{x}) \equiv \tilde{F}(\alpha_i), \qquad g_i(\boldsymbol{x}) \equiv -\alpha_i$$

$$g_{p+i}(\boldsymbol{x}) \equiv \alpha_i - 90 \qquad i = 1, \ldots, p$$

The design variable vector $\boldsymbol{x}$ comprises only fiber orientations α_i, and p is considered as a preassigned parameter. Thus, the buckling load maximization is performed for a fixed mass. As p is the maximum possible number of layers with different orientations in the laminate, this formulation takes maximum advantage of the margin of freedom to affect the buckling load by tailoring the laminate setup. Since global optima have to be found, the potential for reducing the probability of getting merely local optima should be exploited. To perform the maximum search with continuous variation of the design variables, a mathematical programming method has been chosen, which is realized in the CADOP computer program, developed by Pappas.[8] It is based, in the main, on the so-called rotating coordinate pattern search. Pappas reports that this procedure climbs a ridge roughly along the gradient direction and then moves approximately following the mountain chain. It is prepared to clear trivial local extremes and insignificant discontinuities of the objective function. By performing CADOP runs with various distinct starting values for the design variables and by carefully reconsidering the results, the probability is increased to discover the global optimum looked for. Industry frequently prefers designs with fiber orientations of 0, ±45, and 90 deg, i.e., optimization runs with these discrete values of orientations have to be performed, too. For this purpose the method of lattice search is applied. This procedure leads directly to global optima. By that means definite lower limits are available to check the maximum buckling loads, which are based on continuous variation of the fiber orientations. The practically exponential increase of computer time with the number of variables is the price to be paid for the guarantee to obtain global optima.

Results

To begin, buckling load maximizations by variation of fiber orientations were performed for CFRP shells with $R = 250$ mm, $L = 510$ mm, and $t_0 = 0.25$ mm, at any given value of p between 1 and 10. Layer thickness $t_0 = 0.25$ mm means that it is twice the thickness of a tape. This example is called the standard example. Considering continuous variation, six optimization runs with different starting values for the variables were carried out for $p = 1$, whereas for $p = 2$–10 there were generally 12 optimization runs with different starting values. Those orientations that led to the highest buckling load for each p were regarded as the result. As it was not certain whether the global optima have really been found with them, careful evaluation of the optima was performed. If there were indications that they could have been merely local, the optimization was repeated with different starting values. Any optimization with continuous variation was supplemented by an optimization with discrete variation of the design variables ($\alpha_i = 0$ deg or ±45 or 90 deg). The optimum orientations and maximum buckling loads for continuous and discrete variation were recorded in tables such as Tables 1 and 2.

To investigate how the optimum orientations depend on R, L, and t_0, a great many of the computations similar to those for the standard example were carried out with many different combinations of these parameters. The results can be summarized as follows.

1) There are optimum laminates that are close to those of the standard example, as well as optimum laminates which are remarkably different,

2) The maximum buckling loads are practically independent of R and L, apart from those for $p = 2$ and 3 with continuous variation and from those for $p = 3$ and 4 with discrete variation,

3) The buckling loads are proportional to t_0^2 if they are independent of R and L.

It can be shown that, in general, there are laminate setups for which the buckling loads depend on R and L, as well as setups for which this does not hold. If it would be a property of optimum laminates that the respective maximum buckling loads do not depend on R and L, then these laminates would be identical for all of the different combinations of R, L, and t_0. This is confirmed by considering the optimum orientations of the standard example as such orientations and by computation of the respective buckling loads for the many different combinations investigated. Apart from $p = 2$ and 3 for continuous variation, the maximum buckling loads and the buckling loads belonging to the optimum orientations of the standard example differ only by less than 3%. This value is small enough to justify the choice of the optimum orientations of the standard example to be considered also as being practically optimum for all of the other combinations of R, L, and t_0. If we now assume that this result can be generalized to the combinations not investigated also, we are able to define a single table with optimum orientations valid for all of the combinations of R, L, and t_0. To overcome the difficulties with (the not very realistic numbers) $p = 2$ and 3, it is

Table 1 Quick design rule for continuous variation: optimum fiber orientations and maximum buckling loads, t_0 = 0.25 mm

Number of angle plies	Optimum fiber orientations										Classical buckling loads
p	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	$\tilde{F}$, kN
1	14.7										7.2167
2	12.0	32.2									31.395
3	51.0	89.9	33.1								80.329
4	33.3	89.9	0.3	57.3							181.88
5	61.9	0.5	48.3	89.8	29.6						289.39
6	25.1	77.1	60.7	31.6	6.7	66.1					417.02
7	64.8	24.0	17.9	87.7	52.6	45.6	39.0				567.87
8	34.7	88.6	29.6	1.4	63.2	58.6	43.2	37.0			744.48
9	64.4	18.7	69.0	15.4	32.7	45.0	80.2	82.9	3.3		937.86
10	75.2	29.8	26.9	21.9	66.6	61.8	6.5	78.3	77.2	0.3	1158.4

Table 2 Quick design rule for discrete variation: optimum fiber orientations and maximum buckling loads, t_0 = 0.25 mm

Number of angle plies p	Optimum fiber orientations α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	Classical buckling loads $\tilde{F}$, kN
1	45										6.3751
2	0	45									27.356
3	45	90	45								73.942
4	45	0	90	45							159.97
5	45	0	90	0	45						278.18
6	45	0	90	0	45	45					407.62
7	45	0	90	45	0	45	45				558.32
8	45	90	0	45	90	45	45	45			734.39
9	45	0	90	45	0	45	45	45	45		928.08
10	45	90	45	0	0	90	45	90	90	0	1149.9

recommended to consider two different laminate setups as candidates, and to select that with the highest buckling load. Now the rule for quick design with continuous variation reads as follows:

For a given triple of R, L, t_0 and any ply numbers p between 1 and 10, the practically optimum orientations can be taken from Table 1. With $p=2$ and 3 the alternatives $\alpha_1=22.9$ and $\alpha_2=33.5$ deg and $\alpha_1 = 42.7$, $\alpha_2 = 80.7$, and $\alpha_3 = 31.0$ deg, respectively, also have to be considered. The maximum buckling load is obtained by multiplying the buckling load of the table with $(t_0/0.25\text{ mm})^2$ (except for $p = 2$ and 3). The buckling load values hold for the CFRP stiffness properties already given.

The rule for quick design with discrete variation is corresponding; the practically optimum orientations are collated in Table 2. There is no need to consider alternative orientations, but for $p = 3$ and 4 it is not allowed to compute the maximum buckling load using a factor such as that applied in the rule for continuous variation.

The question to be answered is whether these rules are only valid for CFRP laminates with the assumed stiffness properties. There are four independent membrane stiffnesses of a unidirectional ply. This means that from $p = 4$ the design freedom due to membrane stiffness variation could be fully exploited, and an increase of p would not add any more freedom. The freedom due to layering and stacking for continuous and discrete variations is almost completely utilized from $p = 4$ and 5, respectively.[4] Laminates with these numbers of angle plies reveal precisely that design freedom required to approach the state of optimality strived for. The conclusion is that from $p = 4$ (for continuous variation) and 5 (for discrete variation) the optimum orientations of the rules for quick design are valid for any composite material made of consistent unidirectional layers. This is confirmed by optimizations with the given GFRP stiffness properties. The respective buckling loads, however, depend on material properties. The rules are valid within the scope of applicability of classical buckling loads for orthotropic shallow shells.

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